Advanced Wireless Communications

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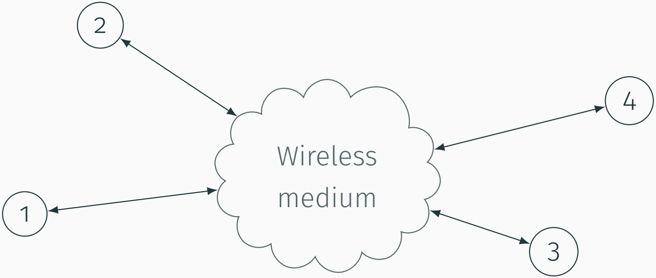
Wireless multiple access generalities

Wireless multiple access

Wireless communications require methods to share the wireless medium.

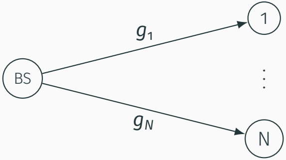


Sharing can be very clean and organized, or completely random—both corresponds to different needs and applications.

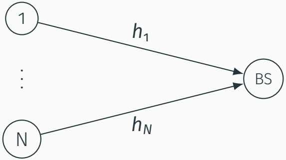
- Clean sharing is usually called multiple access.
- Random sharing is usually called random access.

Wireless multiple access

- Downlink: from base station (BS)/access point (AP) to users



- Uplink: from users to BS/AP



- Channels need not be symmetrical in practice: duplexing is necessary

What is covered

1. Some refresher course on wireless channels and dynamics
2. Multiple access in wireless networks using time or frequency division
3. Non-orthogonal multiple access using superposition
4. Multiple access in wireless networks using multi-antenna techniques
5. Wireless random access networks
6. Massive random access techniques

Each item should take 1-2 hours.

Some preliminary notes

- There will be math in there: learn it
- Everything we cover can be in the exam
- I am not on site; you can contact me at this email

  `paul.r.ferrand@ieee.org`

  or

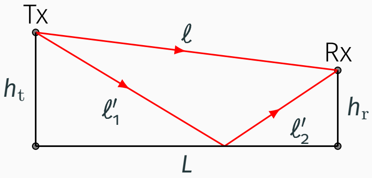
  `paul.ferrand@huawei.com`
- I will recap some things but I assume you had some coverage of
  - Wireless channel models and propagation
  - Orthogonal Frequency Division Multiplexing (OFDM)
  - Multi-antenna communications
  - Information theory
  - Markov chains and queuing theory

Wireless propagation basics

Exercise

1. What is the surface of a sphere with respect to its radius r?
2. Assume that a point antenna radiates equally in all directions. You transmit a wave with initial power P_T. What is the power received at some point at a distance L?
3. How would you write the effect of such path loss on the amplitude of a planar wave $e^{\imath \vec{k}\cdot\vec{x}-\omega t}$?

Large scale propagation

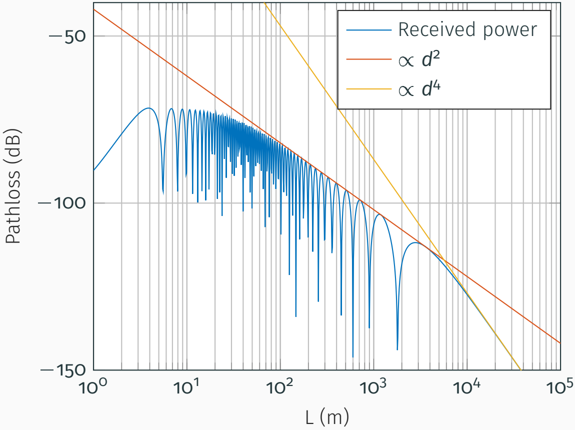


- The reflection is perfect and the reflection coefficient is thus -1.
- L is very much larger than h_r and h_t.
- The amplitude gain of the antenna pair is equal to $\lambda\sqrt{G}$.
- We transmit $s(t)$ onto a planar sinusoidal carrier with pathloss.

Exercise

1. What is the phase difference between the two paths if you transmit a planar sinusoidal carrier? Using $\sqrt{1+x} \approx 1 + x/2$ for small x, find a compact expression.
2. What is the sum power for both impinging waves at the receiver? Using $e^x \approx 1 + x$ for small x, find a compact expression.
3. How does it differ from free space path loss?

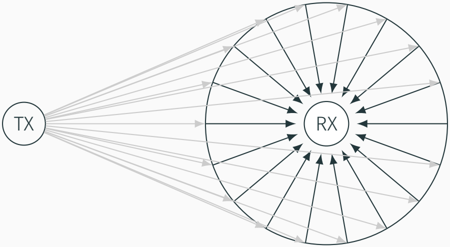
Large scale propagation



In the end, we usually have $P_R = P_T d^{-\alpha}$ for $2 \leq \alpha \leq 4$. We can further add a random shadowing term so that $P_R = S \cdot P_T$ with

$$\log S \sim \mathcal{N}(-\alpha \log d, \sigma_s^2)$$

Small scale propagation

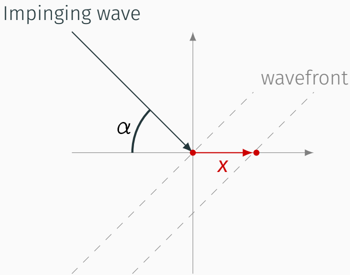


Exercise

Assume that you have N independent impinging rays $h_k = e^{\imath\theta_k}$ on the receiver, each with an i.i.d. random phase $\theta_k \sim \mathcal{U}(-\pi, \pi)$.

1. What is the correct scaling for the rays so that the channel $h = \sum_k h_k$ has unit power?
2. How are the real and imaginary part of k distributed as $N \to \infty$? Do you know the distribution of $|h|$?

Small scale propagation



- The plane wavevector is

$$\vec{k} = \frac{2\pi}{\lambda}\begin{pmatrix}\cos\alpha \\ \sin\alpha\end{pmatrix} = k\begin{pmatrix}\cos\alpha \\ \sin\alpha\end{pmatrix}$$

- The impinging angle α_k is independent of θ_k and uniform.

Exercise

1. What will be the phase offset of the plane wavevector after a small displacement along $\vec{x}$?
2. Define $h(kx)$ as the channel after a small displacement along $\vec{x}$, and define the autocorrelation of the process as $R_h(kx) = \mathbb{E}\left[\bar{h}(0)h(kx)\right]$. Show that

$$R_h(kx) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{\imath kx\cos\alpha}d\alpha = J_0(kx)$$

.

Small scale propagation

Let $\vec{v}$ be the velocity of the user. We can write he small displacement as

$$\vec{x} = \vec{v}\Delta t$$

The value

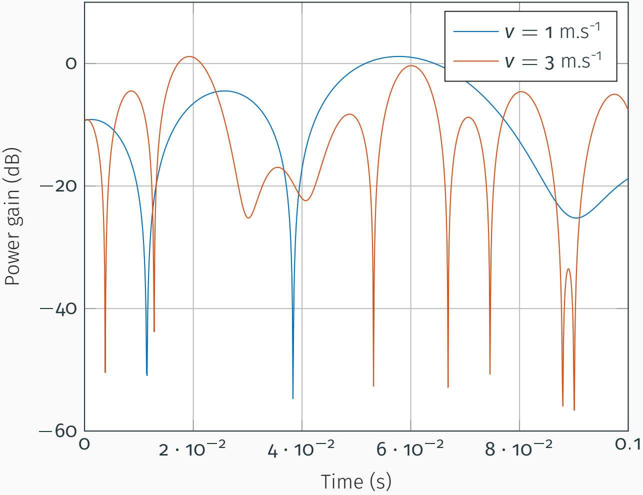
$$\nu_m = k\|v\|$$

is the maximum Doppler frequency of the channel—the worst frequency shift due to motion over a short time.

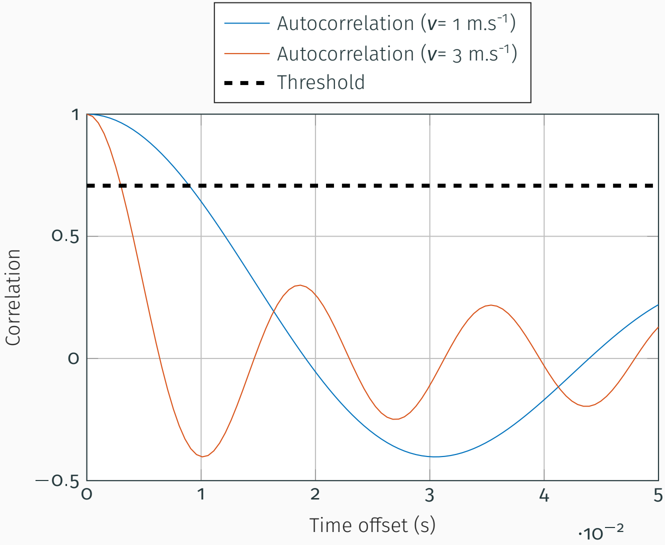
We usually define the coherence time t_c as the value for which $R_h(t_c) = 1/\sqrt{2}$. For our Rayleigh model, we have

$$t_c = \frac{9}{16\pi\nu_m}$$

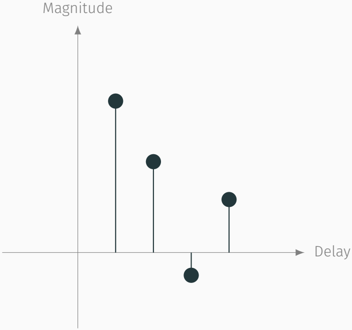
The channel shows very deep fades over time.



Small scale propagation



Small scale propagation



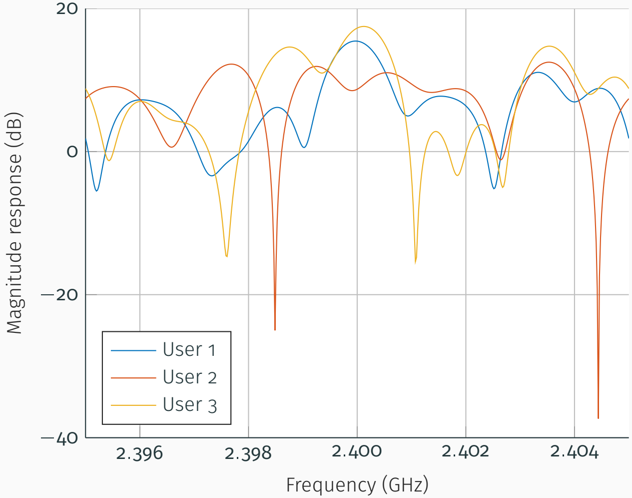
- Assuming uncorrelated scattering you will have independent echos of the signal at different delays
- This can be modeled as a *delay line* channel where each tap is random—e.g. Rayleigh.

Much like the maximum Doppler frequency and the Doppler spread controlled the coherence time of the channel, the delay spread τ_s controls the coherence bandwidth of the channel

$$B_c \propto \frac{1}{\tau_s}$$

Small scale propagation

4 tap Rayleigh channel over 10 MHz bandwidth



How much can we share?

How big is the cake?

For infinite signals with finites energy, we have ($\omega = 2\pi f$)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{\imath\omega t}d\omega \qquad F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-\imath\omega t}dt$$

We want to find the strictly bandlimited signal

$$g(t) = \frac{1}{2\pi}\int_{-W}^{W} F(\omega)e^{\imath\omega t}d\omega$$

that is maximally concentrated in time, so that we maximize

$$\lambda = \frac{\int_{-T}^{T} g^2(t)dt}{\int_{-\infty}^{\infty} g^2(t)dt}$$

How big is the cake?

Slepian *et al.* showed that the spectra for such signal verifies

$$\int_{-W}^{W} \frac{\sin \pi T(\omega - \omega')}{\pi(\omega - \omega')} G(\omega') d\omega' = \lambda G(\omega) \qquad |\omega| \leq W$$

After a change of variable and a scaling, we have

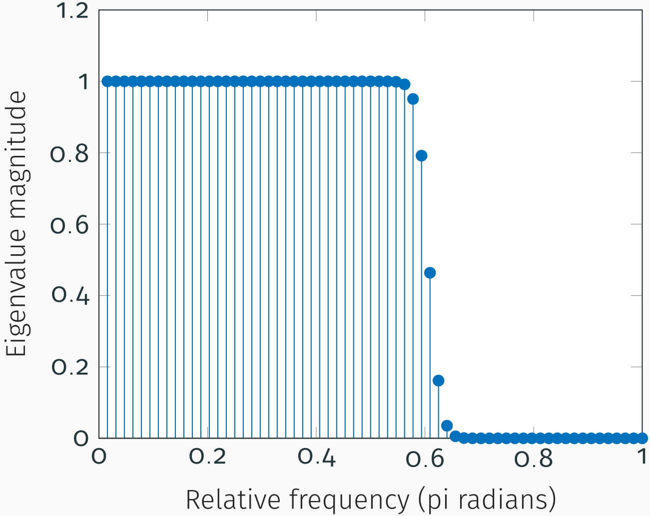
$$\int_{-1}^{1} \frac{\sin 2\pi TW(x - x')}{\pi(x - x')} u(x') dx' = \lambda u(x)$$

Sampling this equation, this leads to

$$\sum_{j=0}^{N-1} \frac{\sin 2\pi W(j - i)}{\pi(j - i)} \vec{u}_i[j] = \lambda_i \vec{u}_i \Longleftrightarrow \boldsymbol{C}(N, W)\vec{u}_i = \lambda_i \vec{u}_i$$

How big is the cake?

Eigenvalue distribution for $\mathbf{C}$ for $T = N = 64$ and $W = 0.3$



How big is the cake?

The eigenvalues fall to 0 after $2TW$, which means that we can safely write that

$$\vec{s}[k] \approx \sum_{i=1}^{2TW} a_i \vec{u}_i[k]$$

or in the continuous time

$$s(t) \approx \sum_{i=1}^{2TW} a_i u_i(t)$$

The size of the space of stricly band-limited, mostly time-limited functions is about $2TW$.

Capacity of a flat channel

The capacity of a band-limited, time-limited and flat channel with amplitude gain h

$$y(t) = hx(t) + z(t) \qquad \mathbb{E}[|x(t)|^2] \leq P$$

is

$$C = TW \log_2 \left(1 + \frac{|h|^2 P}{W N_0}\right) = TW \log_2 \left(1 + \frac{|h|^2 P_w}{N_0}\right)$$

The capacity is linear in frequency and time, and logarithmic in power.

Capacity of a flat channel

Assume discrete time and add an interferer, so that

$$y[k] = hx[k] + x_{\text{interf}}[k] + z[k]$$

with $z[k] \sim \mathcal{N}(0, N_0)$ and $x[k] \sim \mathcal{N}(0, P_I)$. The capacity is then

$$TW \log_2 \left(1 + \frac{|h|^2 P_w}{N_0 + P_I}\right)$$

.

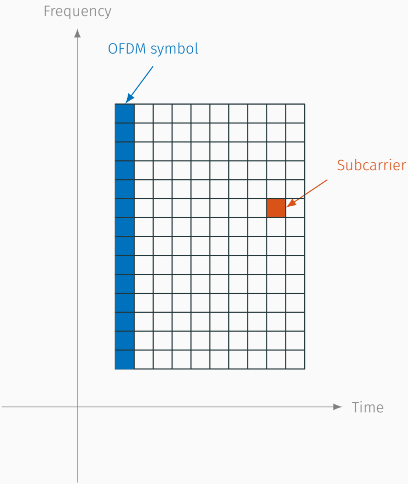
Exercise

1. Consider a user with 2 BS in line of sight. The user is closer to and connected to the first BS. Both transmit at power P_T. Parametrize the large scale channel as necessary and write the capacity of the channel.
2. What happens when P_T increases?

Orthogonal multiple access

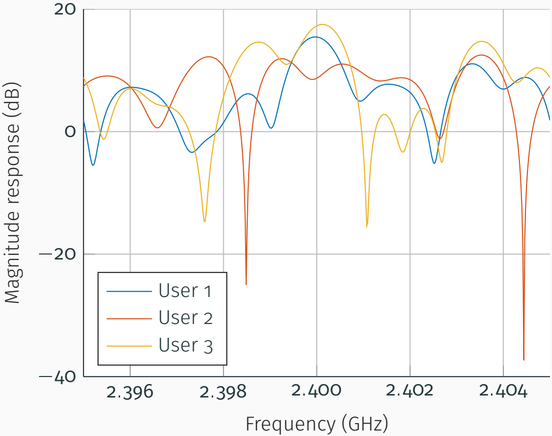
Cutting the time-frequency cake

Enter Orthogonal Frequency Division Multiplexing



- Separates the time-frequency grid into subcarriers/resource elements
- Each subcarrier is a flat channel: the subcarrier spacing is chosen to be less than the coherence bandwidth
- The channel is supposed to be static over an OFDM symbol

Recall the multi-user frequency response for Rayleigh channels: bad subcarriers for each user should not be used for transmission.



Orthogonal Frequency Division Multiple Access

Hypothesis set for the OFDMA system

- Downlink case with K users and N subcarriers
- Normalized channel gain $g_{k,n} = \frac{|h_{k,n}|^2}{N_0}$
- We assume we know the channels
- Capacity per subcarrier $R_{k,n} = W \log_2 (1 + g_{k,n} p_{k,n})$
- Subcarrier set $\Omega = \{1, \ldots, N\}$
- Orthogonal subcarrier sets for users Ω_k with $\bigcup_k \Omega_k = \Omega$
- Constraints on power $p_{k,n} \geq 0$ and $\sum_k \sum_{n \in \Omega_k} p_{k,n} = P_T$

Orthogonal Frequency Division Multiple Access

We want to solve the scheduling problem:

$$\begin{aligned}
\text{Maximize} \quad & f\left(\{\Omega_k\}, \{p_{k,n}\}\right) \\
\text{Subject to} \quad & \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} \leq P_T \\
& p_{k,n} \geq 0 \quad \forall k, n \\
& \Omega_i \cap \Omega_j = \emptyset \quad \text{for } i, j \in \{1, \ldots, K\} \text{ and } i \neq j \\
& \bigcup_k \Omega_k = \Omega
\end{aligned}$$

Exercise

1. Find 2 examples of cost functions for $f\left(\{\Omega_k\}, \{p_{k,n}\}\right)$
2. What constraints are non convex in this problem? How could we relax them?

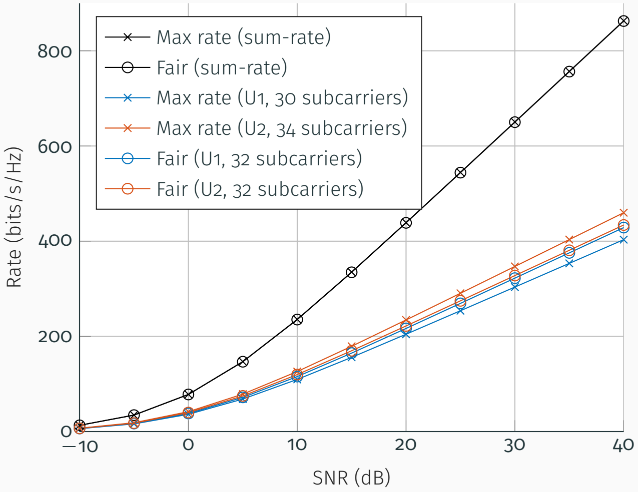
Orthogonal Frequency Division Multiple Access

- We look at greedy algorithms which make local decisions according to a heuristic
- Fix the transmit power $p_{k,n} = P_T/N$ and concentrate on the subcarrier allocation
- A greedy algorithm for the sum-rate maximization may be
    1. Set $\Omega_k = \emptyset$ for all k
    2. For $n = 1, \ldots, N$
        - 2.1 Find the maximum $g_{k',n} \geq g_{k,n}$ for all k
        - 2.2 Set $\Omega_k = \Omega_k \cup \{k'\}$

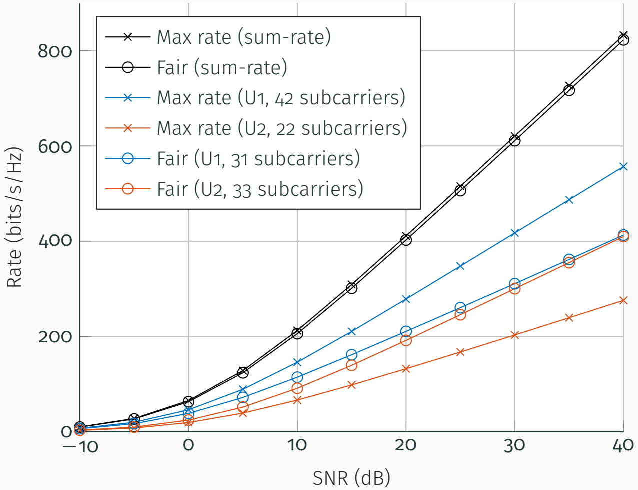
Exercise

1. Considering the heuristic that fair algorithms tend to prioritize low-performing users, design a greedy scheduling algorithm for fairness.

$K = 2$, $N = 64$, Rayleigh channels with equal average gains

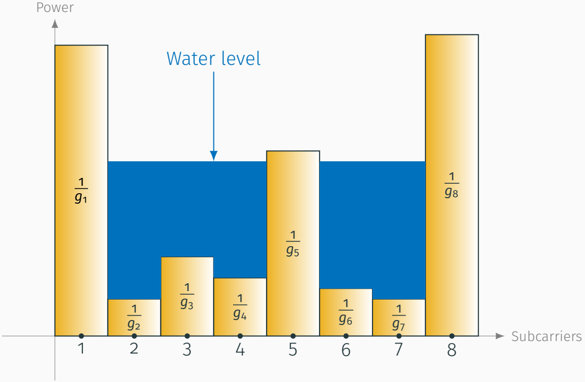


$K = 2$, $N = 64$, Rayleigh channels with 2 dB gain difference



Orthogonal Frequency Division Multiple Access

Maximize the sum-rate under a fixed allocation: waterfilling algorithm



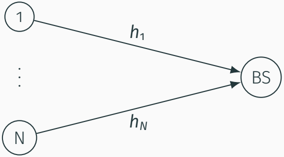
Exercise

1. Waterfilling is not fair: using the previous intuition, write a greedy algorithm which tries to maximize the rates of all subcarriers equally.

Non-orthogonal multiple access

Non-orthogonal multiple access

Uplink scenario



The system equation is

$$y = h_1 x_1 + h_2 x_2 + z$$

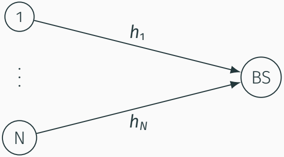
with $\mathbb{E}[|x_1|^2] \leq P_1$, $\mathbb{E}[|x_2|^2] \leq P_2$ and $z \sim \mathcal{N}(0, N_0)$.

Exercise

1. Assuming we want to schedule transmissions for every user, what would be the orthogonal multiple access capacity?
2. What is the sum capacity of the channel? Plot the region of achievable rates less than the capacity using the rate of each user as axes.
3. Consider now that the power constraint P is actually an energy constraint E spent on transmission. What changes?

Non-orthogonal multiple access

Uplink scenario



The system equation is

$$y = h_1 x_1 + h_2 x_2 + z$$

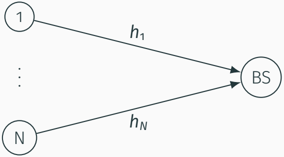
with $\mathbb{E}[|x_1|^2] \leq P_1$, $\mathbb{E}[|x_2|^2] \leq P_2$ and $z \sim \mathcal{N}(0, N_0)$.

Exercise

1. Another option is to allow interference and decode through it. Assume $|h_1|^2 P_1 > |h_2|^2 P_2$. The BS tries to decode the signal from user 1 first, and that x_2 can be assumed $\mathcal{N}(0, P_2)$. What is the capacity of both users?
2. What is their sum capacity? Draw the new achievable rate region.
3. What happens if you try to decode the signal from user 2 first?

Non-orthogonal multiple access

Uplink scenario



The system equation is

$$y = h_1 x_1 + h_2 x_2 + z$$

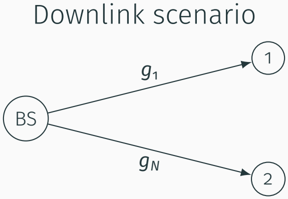
with $\mathbb{E}[|x_1|^2] \leq P_1$, $\mathbb{E}[|x_2|^2] \leq P_2$ and $z \sim \mathcal{N}(0, N_0)$.

We now study an implementation of successive interference cancellation.

Exercise

1. Assume that each user sends signal from a BPSK constellation (i.e. from $\{-1, 1\}$). Let $h_1 = e^{\imath\pi/4}$ and $h_2 = e^{\imath\pi/2}/2$. Draw the constellations through both channels. What shape will have the received signal?
2. What is the minimum distance in the sum constellation? Will the BS be able to support the non-orthogonal multiple access rates through successive cancellation?
3. What if user 1 uses a QPSK constellation? What if both users use a QPSK constellation?

Non-orthogonal multiple access



Exercise

1. Write down the system model.
2. What simple medium sharing method can you implement?
3. Can you think of an analog to successive interference cancellation in this case?

Multiple access with multiple antennas

Matched filtering

Consider the following single input, multiple output (SIMO) system

$$\vec{y} = \vec{h}\sqrt{P}x + \vec{z}$$

Assume $\vec{h}$ is known, $\mathbb{E}\left[\vec{z}\vec{z}^H\right] = \boldsymbol{I}$, $||\vec{h}|| = 1$ and $\mathbb{E}[|x|^2] = 1$.

Exercise

We want to find $\vec{w}$ so that $\hat{x} = \vec{w}^H\vec{y}$.

1. Write down the signal to noise ratio of the detection using a generic $\vec{w}$.
2. What would be the choice of $\vec{w}$ that maximizes this SNR?
3. What would happen in a MISO system?

Matched filtering

Now consider 2 users multiplexed in the system

$$\vec{y} = \vec{h}_1\sqrt{P_1}x_1 + \vec{h}_2\sqrt{P_2}x_2 + \vec{z}$$

Exercise

1. Using the matched filtered solution of the previous case, what would be the SNRs of users 1 and 2?
2. Assuming $P_1 = P_2 = P$, how does the SNR behave as $P \to \infty$?
3. Write the system and detector in matrix form.
4. Can you think of a different approach to alleviate the scaling problem?

We still consider 2 users multiplexed in the system

$$\vec{y} = \vec{h}_1\sqrt{P_1}x_1 + \vec{h}_2\sqrt{P_2}x_2 + \vec{z}$$

Exercise

1. Check that $\boldsymbol{\Pi}_2 = \boldsymbol{I} - \vec{h}_2\vec{h}_2^H$ verifies $\boldsymbol{\Pi}_2\vec{h}_2 = \vec{0}$.
2. Consider the new system formed with $\vec{y}' = \boldsymbol{\Pi}_2\vec{y}$. What is the optimal choice of weights $\vec{w}_1$ to estimate x_1 in this new system?
3. Using vector calculus, find $\vec{x}$ so that the least-square objective is minimized

$$\|\vec{y} - \boldsymbol{H}\vec{x}\|^2 = (\vec{y} - \boldsymbol{H}\vec{x})^H(\vec{y} - \boldsymbol{H}\vec{x})$$

Minimum mean-square error

We still consider 2 users multiplexed in the system

$$\vec{y} = \vec{h}_1 \sqrt{P_1} x_1 + \vec{h}_2 \sqrt{P_2} x_2 + \vec{z}$$

Assume that each x_1, x_2 and z are i.i.d. $\mathcal{CN}(0, 1)$. Expectations are over the ensembles of x_1, x_2 and z.

Exercise

1. What is the covariance $\boldsymbol{C}_{\vec{y}} = \mathbb{E}\left[\vec{y}\vec{y}^H\right]$ of $\vec{y}$?
2. What is the cross-covariance $\boldsymbol{C}_{\vec{y}x_i} = \mathbb{E}\left[\vec{y}x_i\right]$ between $\vec{y}$ and x_i?
3. Find the weight vector that minimizes the residual MSE

$$\mathbb{E}\left[|\vec{w}^H\vec{y} - x_1|^2\right]$$

4. How can you write the weight matrix that minimizes the residual MSE for both x_1 and x_2?

We still consider 2 users multiplexed in the system

$$\vec{y} = \vec{h}_1 \sqrt{P_1} x_1 + \vec{h}_2 \sqrt{P_2} x_2 + \vec{z}$$

with $\vec{h}_i = \sqrt{s_i}(h_{i,1}, \cdots, h_{i,N})^T$ and i.i.d. $h_{i,n} \sim \mathcal{N}(0, 1)$.

Exercise

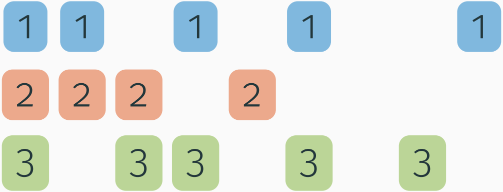
The key idea now is to let the number of antennas increase.

1. Considering that we don't have $\|h_i\|^2 = 1$, what is going to be the pseudo-SNR of the matched filters in this case?
2. Using the weak law of large numbers, compute $\frac{1}{N}\|\vec{h}_i\|^2$ and $\frac{1}{N}\vec{h}_j^H \vec{h}_i$ for $i \neq j$ as the number of antennas N grows without bound.
3. What can you conclude?

Random access

The ALOHA protocols

What if we allowed possible collisions?



The *slotted* ALOHA protocol:

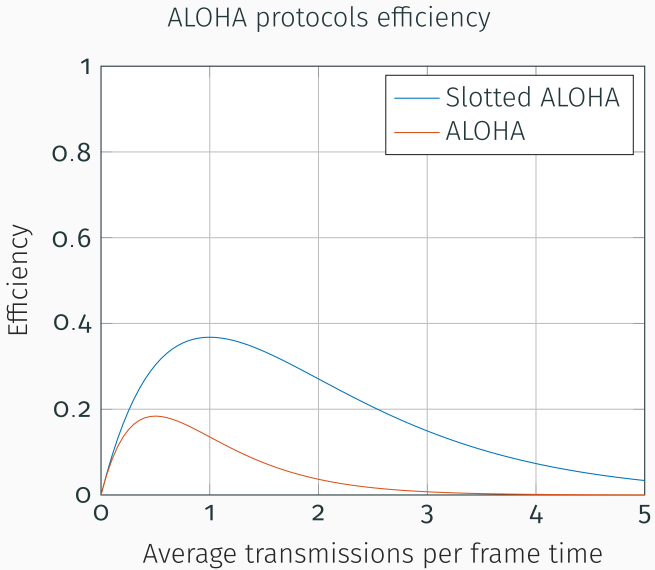
- All nodes are synchronized on transmission slots
- Everyone can detect collisions
- Each node sends data immediately; if a collision occurs, it sends it again with probability p in subsequent slots

The ALOHA protocols

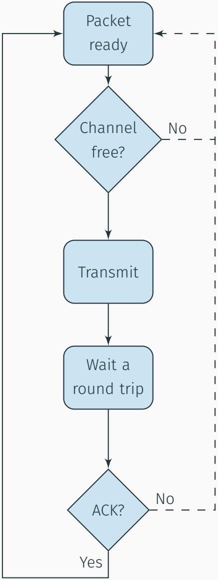
Each node sends data immediately; if a collision occurs, it sends it again with probability p in subsequent slots

Exercises

1. Suppose there are N nodes in the network. What is the success probability of a single node?
2. What is then the efficiency overall in the network?
3. How can we optimize this efficiency?
4. Assume now that nodes may start transmitting uniformly inside a slot while still having a probability p of transmitting in a slot. How is the efficiency evolving? (Note : this is the original ALOHA)
5. What is the maximum efficiency in both cases?

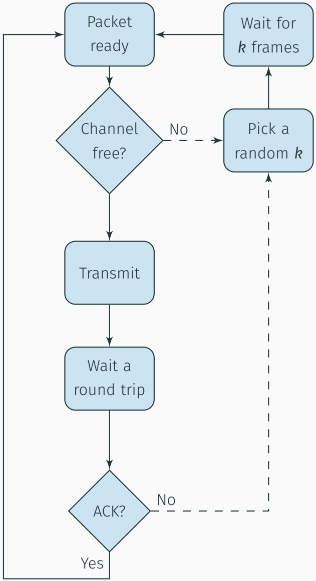

ALOHA protocols efficiency
Slotted ALOHA
ALOHA
Efficiency
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
Average transmissions per frame time

Carrier-sensing



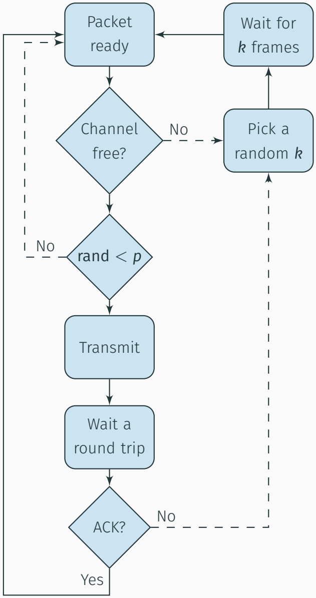
- CSMA is less greedy: check if someone uses the channel
- Collisions when 2 nodes are waiting for the current transmission to end

Carrier-sensing multiple access

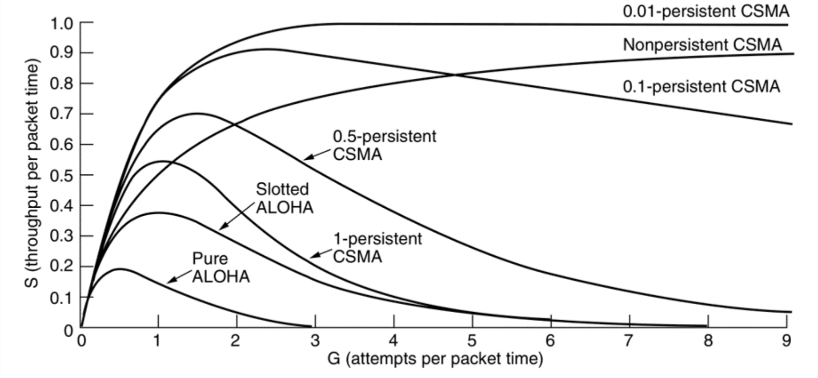


- CSMA is less greedy: check if someone uses the channel
- Collisions when 2 nodes are waiting for the current transmission to end
- Non-persistence: add a random backoff before sensing again

Carrier-sensing multiple access

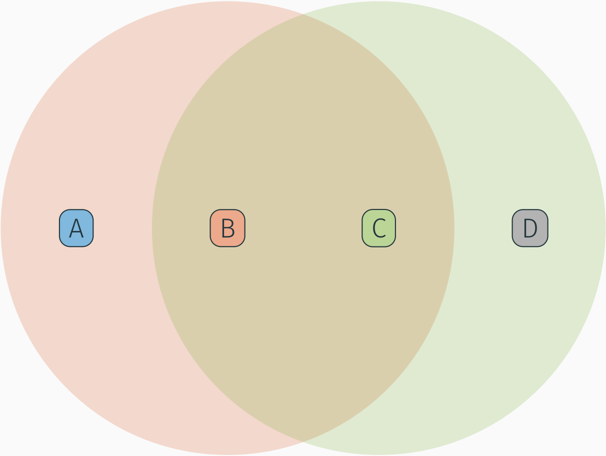


- CSMA is less greedy: check if someone uses the channel
- Collisions when 2 nodes are waiting for the current transmission to end
- Non-persistence: add a random backoff before sensing again
- p-persistence: add a transmission probability



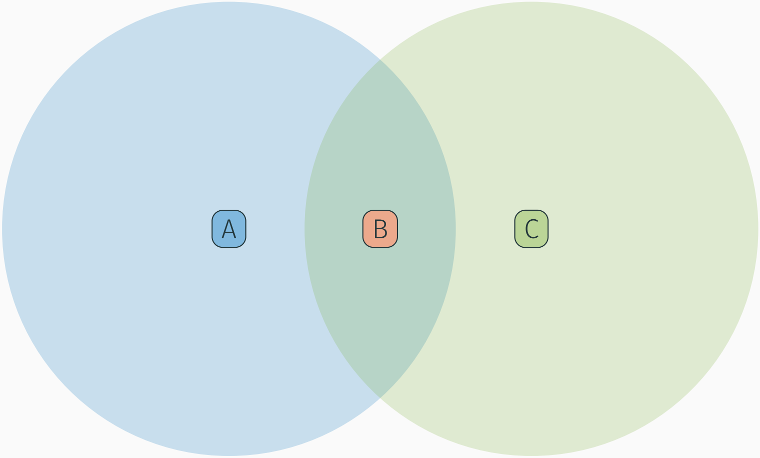
From "Computer Networks", A. Tanenbaum

How to avoid close range interference (exposed terminal)?



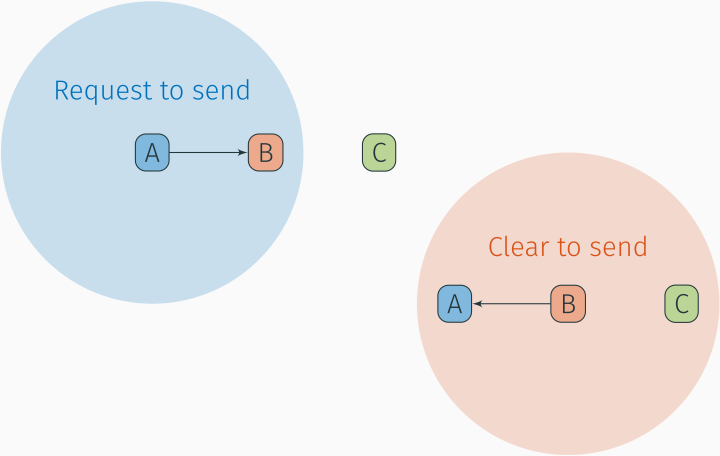
Carrier-sensing in Wireless

How to detect collisions for out-of-range nodes (hidden terminal)?



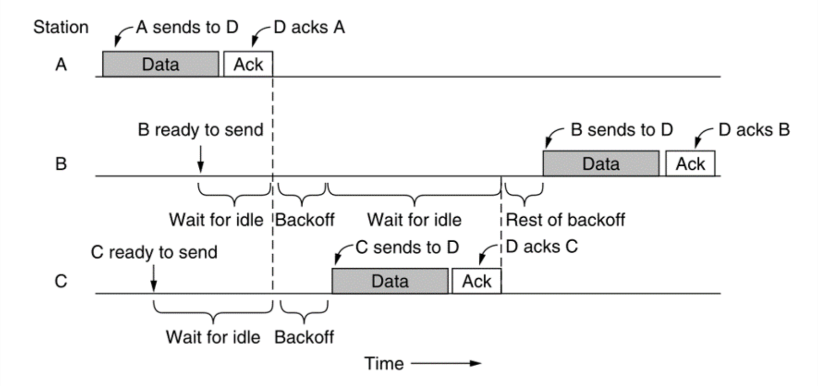
Carrier-sensing in wireless

Emulate the carrier sensing at the receiver with specific packets



CSMA/CA : collision avoidance

Draw a random backoff before the transmission



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